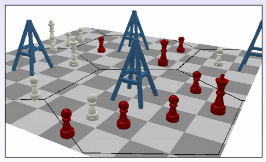
# Game Theory in the Analysis and Design of Cognitive Radio Networks

James Neel
James.neel@crtwireless.com
(540) 230-6012
www.crtwireless.com



DySPAN 2007
April 17, 2007

CRT
Cognitive Radio Technologies



# CRT Information

Small business officially incorporated in Feb 2007 to commercialize cognitive radio research

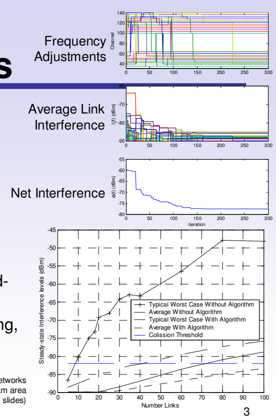
Email: james.neel@crtwireless.com
reedjh@crtwireless.com
Website: crtwireless.com
Tel: 540-230-6012
Mailing Address:
Cognitive Radio Technologies, LLC
147 Mill Ridge Rd, Suite 119
Lynchburg, VA 24502





# CRT's Strengths

- Analysis of networked cognitive radio algorithms (game theory)
- Design of low complexity, low overhead (scalable), convergent and stable cognitive radio algorithms
  - Infrastructure, mesh, and ad-hoc networks
  - DFS, TPC, AIA, beamforming, routing, topology formation




Statistics from yet to be published DFS algorithm for ad-hoc networks
P2P 802.11a links in 0.5kmx0.5km area
(discussed in tomorrow's slides)





# Tutorial Background

- Most material from my three week defense
  - Very understanding committee
  - Dissertation online @ http://scholar.lib.vt.edu/theses/available/etd-12082006-141855/
  - Original defense slides @ http://www.mprg.org/people/gametheory/Meetings.shtml
- Other material from training short course I gave in summer 2003
  - http://www.mprg.org/people/gametheory/Class.shtml
- Eventually will be formalized into a book





# Approximate Schedule

| Time | Material |
|---|---|
| 1:30-2:15 | Cognitive Radio and Game Theory (51) |
| 2:15-3:00 | Steady-state Solution Concepts (38) |
| 3:00-3:15 | Performance Metrics (11) |
| 3:15-3:30 | Break |
| 3:30-4:10 | Notion of Time and Imperfections in Games (34) |
| 4:10-4:45 | Using Game Theory to Design Cognitive Radio Networks (28) |
| 4:45-5:00 | Summary (14) |





# General Comments on Tutorial

- "This talk is intended to provide attendees with knowledge of the most important game theoretic concepts employed in state-of-the-art dynamic spectrum access networks."
- Lots of concepts, no proofs – cramming 2-3 semesters of game theory into 3.5 hours
- Tutorial can provide quick reference for concepts discussed at conference
- More leisurely sources of information:
  - D. Fudenberg, J. Tirole, *Game Theory*, MIT Press 1991.
  - R. Myerson, *Game Theory: Analysis of Conflict*, Harvard University Press, 1991.
  - M. Osborne, A. Rubinstein, *A Course in Game Theory*, MIT Press, 1994.
  - J. Neel. J. Reed, A. MacKenzie, *Cognitive Radio Network Performance Analysis* in **Cognitive Radio Technology**, B. Fette, ed., Elsevier August 2006.

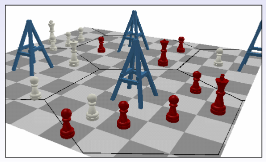
 Image modified from http://hacks.mit.edu/Hacks/by_year/1991/fire_hydrant/

# Cognitive Radio and Game Theory

Cognitive Radio,

Game Theory,

Relationship Between the Two







# Basic Game Concepts and Cognitive Radio Networks

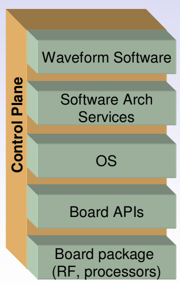
- Assumptions about Cognitive Radios and Cognitive Radio Networks
  - Definition and concept of cognitive radio as used in this presentation
  - Design Challenges Posed by Cognitive Radio Networks
  - A Model of a Cognitive Radio Network
- High Level View of Game Theory
  - Common Components
  - Common Models
- Relationship between Game Theory and Cognitive Radio Networks
  - Modeling a Generic Cognitive Radio Network as a Game
  - Differences in Typical Assumptions
  - Limitations of Application





# Cognitive Radio: Basic Idea

- Software radios permit network or user to control the operation of a software radio
- Cognitive radios enhance the control process by adding
  - Intelligent, autonomous control of the radio
  - An ability to sense the environment
  - Goal driven operation
  - Processes for learning about environmental parameters
  - Awareness of its environment
    - Signals
    - Channels
  - Awareness of capabilities of the radio
  - An ability to negotiate waveforms with other radios








# Conceptual Operation

**OODA Loop:** (continuously)

- Observe outside world
- Orient to infer meaning of observations
- Adjust waveform as needed to achieve goal
- Implement processes needed to change waveform

**Other processes:** (as needed)

- Adjust goals (Plan)
- Learn about the outside world, needs of user,…

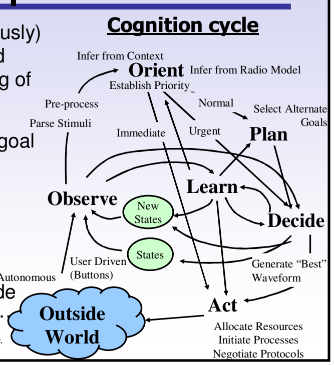
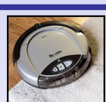



Figure adapted From Mitola, "Cognitive Radio for Flexible Mobile Multimedia Communications ", IEEE Mobile Multimedia Conference, 1999, pp 3-10.

# Implementation Classes



- **Weak** cognitive radio
  - Radio's adaptations determined by hard coded algorithms and informed by observations
  - Many may not consider this to be cognitive (see discussion related to Fig 6 in 1900.1 draft)

- **Strong** cognitive radio
  - Radio's adaptations determined by conscious reasoning
  - Closest approximation is the ontology reasoning cognitive radios

- In general, strong cognitive radios have potential to achieve both much better and much worse behavior in a network, but may not be realizable.

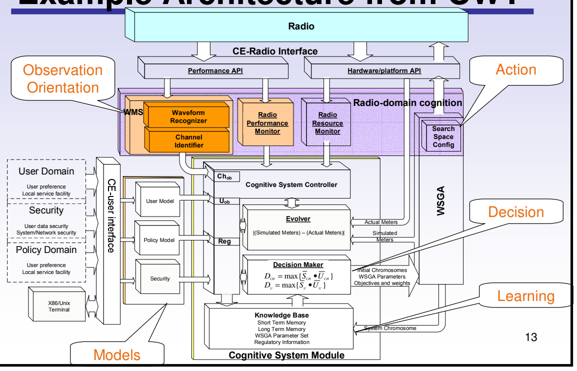




# Brilliant Algorithms and Cognitive Engines

- Most research focuses on development of algorithms for:
  - Observation
  - Decision processes
  - Learning
  - Policy
  - Context Awareness
- Some complete OODA loop algorithms
- In general different algorithms will perform better in different situations
- Cognitive engine can be viewed as a software architecture
- Provides structure for incorporating and interfacing different algorithms
- Mechanism for sharing information across algorithms
- No current implementation standard

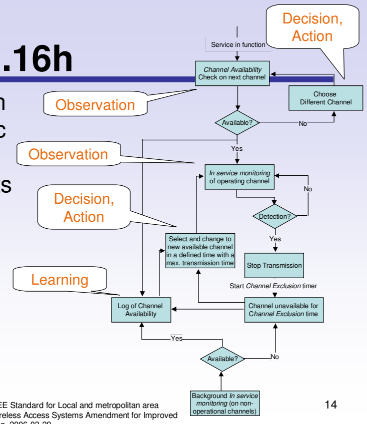
## Example Architecture from CWT






## DFS in 802.16h

- Drafts of 802.16h defined a generic DFS algorithm which implements observation, decision, action, and learning processes
- Very simple implementation




Modified from Figure h1 IEEE 802.16h-06/010 Draft IEEE Standard for Local and metropolitan area networks Part 16: Air Interface for Fixed Broadband Wireless Access Systems Amendment for Improved Coexistence Mechanisms for License-Exempt Operation, 2006-03-29



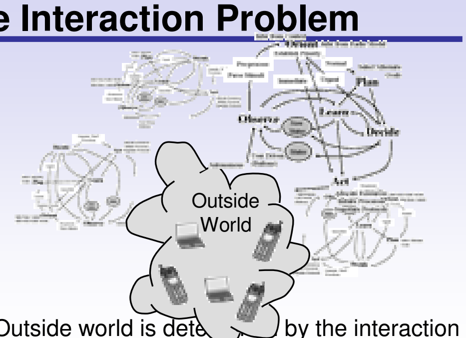
## Used cognitive radio definition

- A *cognitive radio* is a radio whose control processes permit the radio to leverage situational knowledge and *intelligent* processing to autonomously adapt towards some goal.
- Intelligence as defined by [American Heritage_00] as "*The capacity to acquire and apply knowledge, especially toward a **purposeful goal**.*"
  - To eliminate some of the mess, I would love to just call cognitive radio, "intelligent" radio, i.e.,
  - a radio with the capacity to acquire and apply knowledge especially toward a purposeful goal





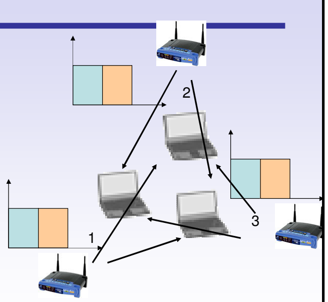
## The Interaction Problem




- Outside world is dete... by the interaction of numerous cognitive radios
- Adaptations spawn adaptations

## Dynamic Spectrum Access Pitfall

- Suppose
  - $g_{31}>g_{21}; g_{12}>g_{32}; g_{23}>g_{13}$
- Without loss of generality
  - $g_{31}, g_{12}, g_{23} = 1$
  - $g_{21}, g_{32}, g_{13} = 0.5$
- Infinite Loop!
  - 4,5,1,3,2,6,4,…




Interference Characterization

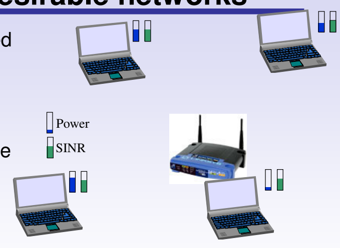
| Chan. | (0,0,0) | (0,0,1) | (0,1,0) | (0,1,1) | (1,0,0) | (1,0,1) | (1,1,0) | (1,1,1) |
|---|---|---|---|---|---|---|---|---|
| Interf. | (1.5,1.5,1.5) | (0.5,1,0) | (1,0,0.5) | (0,0.5,1) | (0,0.5,1) | (1,0,0.5) | (0.5,1,0) | (1.5,1.5,1.5) |
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |

## Implications

- In one out every four deployments, the example system will enter into an infinite loop
- As network scales, probability of entering an infinite loop goes to 1:
  - 2 channels $p(loop) \geq 1-(3/4)^{{}^{n}C_{3}}$
  - k channels $p(loop) \geq 1-\left(1-2^{-k+1}\right)^{{}^{n}C_{k+1}}$
- Even for apparently simple algorithms, ensuring convergence and stability will be nontrivial

## Locally optimal decisions that lead to globally undesirable networks

- Scenario: Distributed SINR maximizing power control in a single cluster
- For each link, it is desirable to increase transmit power in response to increased interference
- Steady state of network is all nodes transmitting at maximum power




**Insufficient to consider only a single link, must consider interaction**



## Potential Problems with Networked Cognitive Radios

**Distributed**

- Infinite recursions
- Instability (chaos)
- Vicious cycles
- Adaptation collisions
- Equitable distribution of resources
- Byzantine failure
- Information distribution

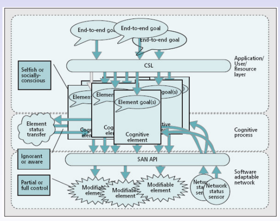
**Centralized**

- Signaling Overhead
- Complexity
- Responsiveness
- Single point of failure




## Cognitive Networks

- Rather than having intelligence reside in a single device, intelligence can reside in the network
- Effectively the same as a centralized approach
- Gives greater scope to the available adaptations
  - Topology, routing
  - Conceptually permits adaptation of core and edge devices
- Can be combined with cognitive radio for mix of capabilities
- Focus of E$^2$R program



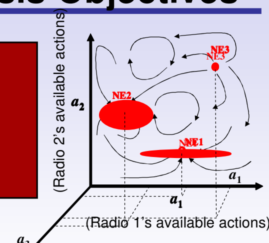
R. Thomas et al., "Cognitive networks: adaptation and learning to achieve end-to-end performance objectives," IEEE Communications Magazine, Dec. 2006




## Network Analysis Objectives

focus

1. Steady state characterization
2. Steady state optimality
3. Convergence
4. Stability/Noise
5. Scalability

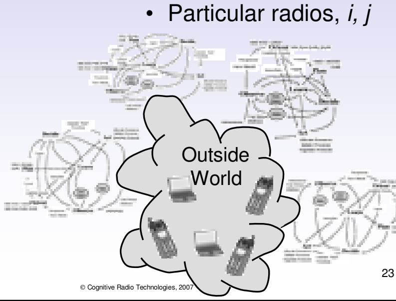



**Steady State Characterization**



## General Model (Focus on OODA Loop Interactions)

- Cognitive Radios
- Set *N*
- Particular radios, *i, j*



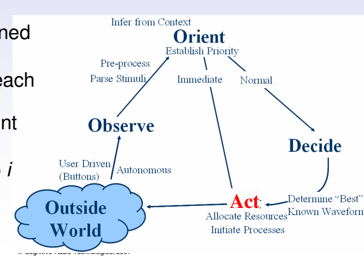




## General Model (Focus on OODA Loop Interactions)

**Actions**

- Different radios may have different capabilities
- May be constrained by policy
- Should specify each radio's available actions to account for variations
- Actions for radio *i*
  - $A_i$

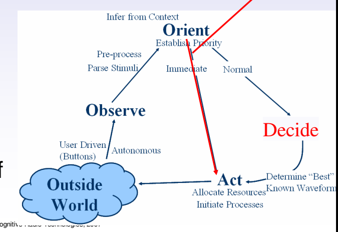


## General Model (Focus on OODA Loop Interactions)

**Decision Rules**

- Maps observations to actions
  - $d_i:O\rightarrow A_i$
- Intelligence implies that these actions further the radio's goal
  - $u_i:O\rightarrow\mathbb{R}$
- Interesting problem: simultaneously modeling behavior of ontological and procedural radios

Implies very simple, deterministic function, e.g., standard interference function






## Cognitive Radio Network Modeling Summary

- Decision making radios — $i,j \in N$, $|N| = n$
- Actions for each radio — $A=A_1\times A_2\times\cdots\times A_n$
- Observed Outcome Space — $O$
- Goals — $u_j:O\rightarrow\mathbb{R}$ ($u_j:A\rightarrow\mathbb{R}$)
- Decision Rules — $d_j:O\rightarrow A_i$ ($d_j:A\rightarrow A_i$)
- Timing — $T=T_1\cup T_2\cup\cdots\cup T_n$

| Symbol | Meaning | Symbol | Meaning |
|---|---|---|---|
| $N$ | Set of cognitive radios | $i, j$ | Particular cognitive radios |
| $A_j$ | Adaptations for $j$ | $a_j$ | Adaptation chosen by $j$ |
| $a_{-j}$ | Adaptation vector excluding $a_j$ | $u_j$ | Goal of $j$ |
| $O$ | Set of outcomes | $O_j$ | Outcome observed by $j$ |
| $d_j$ | Decision rule for $j$ | $T_j$ | Times when $j$ adapts |
| $T$ | Adaptation times $\forall j\in N$ | $t$ | An element of $T$ |

## Comments on Timing

- When decisions are made also matters and different radios will likely make decisions at different time
- $T_j$ – when radio $j$ makes its adaptations
  - Generally assumed to be an infinite set
  - Assumed to occur at discrete time
    - Consistent with DSP implementation
- $T=T_1\cup T_2\cup\cdots\cup T_n$
- $t\in T$

Decision timing classes

- Synchronous
  - All at once
- Round-robin
  - One at a time in order
  - Used in a lot of analysis
- Random
  - One at a time in no order
- Asynchronous
  - Random subset at a time
  - Least overhead for a network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## Basic Game Components

1. A (well-defined) set of 2 or more players
2. A set of actions for each player.
3. A set of preference relationships for each player for each possible action tuple.

- More elaborate games exist with more components but these three must always be there.
- Some also introduce an outcome function which maps action tuples to outcomes which are then valued by the preference relations.
- Games with just these three components (or a variation on the preference relationships) are said to be in **Normal** form or **Strategic** Form


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## Set of Players (decision makers)

- $N$ – set of $n$ players consisting of players "named" {1, 2, 3,…,$i$, $j$,…,$n$}
- Note the $n$ does not mean that there are 14 players in every game.
- Other components of the game that "belong" to a particular player are normally indicated by a subscript.
- Generic players are most commonly written as $i$ or $j$.
- Usage: $N$ is the SET of players, $n$ is the number of players.
- $N \setminus i$ = {1,2,…,$i$-1, $i$+1 ,…, $n$} All players in $N$ except for $i$


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## Actions

$A_i$ – Set of available actions for player $i$

$a_i$ – A particular action chosen by $i$, $a_i\in A_i$

$A$ – Action Space, Cartesian product of all $A_i$

$A=A_1\times A_2\times\cdots\times A_n$

$a$ – Action tuple – a point in the Action Space

$A_{-i}$ – Another action space $A$ formed from

$A_{-i}=A_1\times A_2\times\cdots\times A_{i-1}\times A_{i+1}\times\cdots\times A_n$

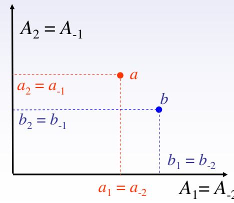
$a_{-i}$ – A point from the space $A_{-i}$

$A = A_i\times A_{-i}$

Example Two Player Action Space

$A_1 = A_2 = [0\ \infty)$

$A=A_1\times A_2$



## Preference Relations (1/2)

Preference Relation expresses an individual player's desirability of one outcome over another (A binary relationship)

$\succsim_i$ Preference Relationship (prefers at least as much as)

$o \succsim_i o^*$ $o$ is preferred at least as much as $o^*$ by player $i$

$\succ_i$ Strict Preference Relationship (prefers strictly more than)

$o \succ_i o^*$ iff $o \succsim_i o^*$ but not $o^* \succsim_i o$

$\sim_i$ "Indifference" Relationship (prefers equally)

$o \sim_i o^*$ iff $o \succsim_i o^*$ and $o^* \succsim_i o$





## Preference Relations (2/2)

- Games generally assume the relationship between actions and outcomes is invertible so preferences can be expressed over action vectors.
- Preferences are really an *ordinal* relationship
  - Know that player prefers one outcome to another, but quantifying by how much introduces difficulties





## Utility Functions (1/2) (Objective Fcns, Payoff Fcns)

A mathematical description of preference relationships.

Maps action space to set of real numbers.

$u_i : A \rightarrow \mathbb{R}$

Preference Relation then defined as

$a \succsim_i a^*$ iff $u_i(a) \geq u_i(a^*)$

$a \succ_i a^*$ iff $u_i(a) > u_i(a^*)$

$a \sim_i a^*$ iff $u_i(a) = u_i(a^*)$





## Utility Functions (2/2)

By quantifying preference relationships all sorts of valuable mathematical operations can be introduced.

Also note that the quantification operation is not unique as long as relationships are preserved. Many map preference relationships to [0,1].

**Example**

Jack prefers Apples to Oranges

$Apples \succ_{Jack} Oranges \iff u_{Jack}(Apples) > u_{Jack}(Oranges)$

a) $u_{Jack}(Apples) = 1$, $u_{Jack}(Oranges) = 0$

b) $u_{Jack}(Apples) = -1$, $u_{Jack}(Oranges) = -7.5$





## Normal Form Games (Strategic Form Games)

In normal form, a game consists of three primary components

$G = \langle N, A, \{u_i\} \rangle$

$N$ – Set of Players

$A_i$ – Set of Actions Available to Player $i$

$A$ – Action Space $A = A_1 \times A_2 \times \cdots \times A_n$

$\{u_i\}$ – Set of Individual Objective Functions

$u_i : A \rightarrow \mathbb{R}$





## Normal Formal Games in Matrix Representation

Useful for representing 2 player games with finite action sets.

Player 1's actions are indexed by rows.

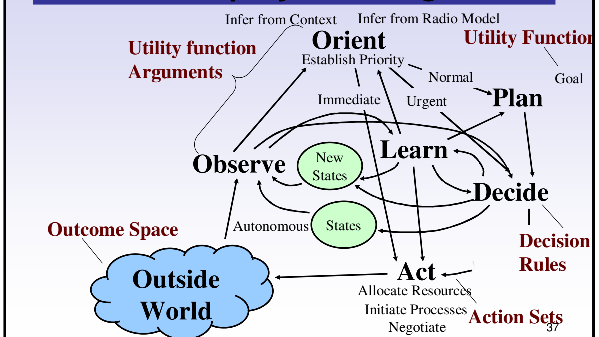
Player 2's actions are indexed by columns.

Each entry is the payoff vector, $(u_1, u_2)$, corresponding to the action tuple

$N = \{1,2\}$ $A_1 = \{a_1, b_1\}$ $A_2 = \{a_2, b_2\}$

| | $a_2$ | $b_2$ |
|---|---|---|
| $a_1$ | $u_1(a_1, a_2), u_2(a_1, a_2)$ | $u_1(a_1, b_2), u_2(a_1, b_2)$ |
| $b_1$ | $u_1(b_1, a_2), u_2(b_1, a_2)$ | $u_1(b_1, b_2), u_2(b_1, b_2)$ |

## Cognitive radios are naturally modeled as players in a game

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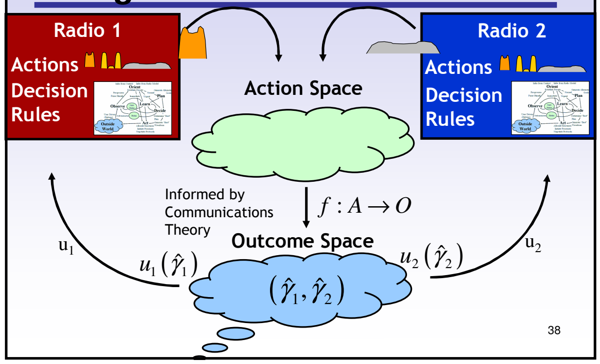
Adapted From Mitola, "Cognitive Radio for Flexible Mobile Multimedia Communications ", IEEE Mobile Multimedia Conference, 1999, pp 3-10.

## Interaction is naturally modeled as a game

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## When Game Theory can be Applied

| Level |
|---|
| 0 SDR |
| 1 Goal Driven |
| 2 Context Aware |
| 3 Radio Aware |
| 4 Planning |
| 5 Negotiating |
| 6 Learns Environment |
| 7 Adapts Plans |
| 8 Adapts Protocols |

**If distributed adaptation doesn't occur, it's not a game** (Level 0)

**Suitable for game theory analysis** (Levels 1–6)

**Unconstrained action sets (radios can make up new adaptations) or undefined goals (utility functions) make analysis impractical** (Levels 7–8)

**Game Theory applies to: 1. Adaptive aware radios**
**2. Cognitive radios that learn about their environment**

## Conditions for Applying Game Theory to CRNs

- Conditions for rationality
  - Well defined decision making processes
  - Expectation of how changes impacts performance
- Conditions for a nontrivial game
  - Multiple interactive decision makers
  - Nonsingleton action sets
- Conditions generally satisfied by distributed dynamic CRN schemes

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## Example Application Appropriateness

- Inappropriate applications
  - Cellular Downlink power control (single cell)
  - Site Planning
  - A single cognitive network
- Appropriate applications
  - Multiple interactive cognitive networks
  - Distributed power control on non-orthogonal waveforms
    - Ad-hoc power control
    - Cell breathing
  - Adaptive MAC
  - Distributed Dynamic Frequency Selection
  - Network formation (localized objectives)

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## Some differences between game models and cognitive radio network model

- Assuming numerous iterations, normal form game only has a single stage.
  - Useful for compactly capturing modeling components at a single stage
  - Normal form game properties will be exploited in the analysis of other games
  - Other game models discussed throughout this presentation

| | Player | Cognitive Radio |
|---|---|---|
| Knowledge | Knows *A* | Can learn *O* (may know or learn *A*) |
| $f : A \rightarrow O$ | Invertible<br>Constant<br>Known | Not invertible (noise)<br>May change over time (though relatively fixed for short periods)<br>Has to learn |
| Preferences | Ordinal | Cardinal (goals) |

# Summary

- Adaptations of cognitive radios interact
  - Adaptations can have unexpected negative results
    - Infinite recursions, vicious cycles
  - Insufficient to consider behavior of only a single link in the design
- Behavior of collection of radios can be modeled as a game
- Some differences in models and assumptions but high level mapping is fairly close

| Game | ⇔ | Cognitive radio network |
|---|---|---|
| Player | ⇔ | Cognitive radio |
| Actions | ⇔ | Actions |
| Utility function | ⇔ | Goal |
| Outcome space | ⇔ | Outside world |
| Utility function arguments | ⇔ | Observations/orientation |
| Order of play | ⇔ | Adaptation timings |

- As we look at convergence, performance, collaboration, and stability, we'll extend the model

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